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An algorithm for joint location, coverage and routing in wireless sensor networks

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Abstract: *This paper deals with selected joint problem of location, coverage and routing in a class of wireless sensor networks. The minimization of the total cost of data collection and transmission as well as sensors and sinks location is considered. Its NP-hardness is justified and a heuristic solution algorithm based on the result of the circulation problem in a directed graph is proposed. The quality of the algorithm has been assessed during numerical experiments, and the examples of corresponding results are presented.*

Keywords: *location, coverage, routing, wireless sensor networks, circulation*

1. Introduction

A progress in new optimization methods and algorithms as well as in corresponding computing tools makes it possible to investigate more complex problems which are closer to real world applications. It concerns operations research problems, in general, and combinatorial problems, in particular, where complex optimization problems, being the combination of interrelated, known and separately developed sub-problems, are intensively studied.

Such complex optimization problems can be encountered in logistic systems (see e.g. [7],[16]), computer networks (see e.g. [19], [2]) and sensor networks which are the subject of discussion in this paper. As examples of such complex problems one can mention: location-routing problem ([22], [24]), location-scheduling problem ([11], [15]), inventory-location problem ([8]), inventory-routing problem ([18], [20]), routing-scheduling problem ([13],[14],[21]), production-inventory problem ([3]), production-transportation problem ([9]).

The extension of this approach for relatively new applications, namely for wireless sensor networks (WSNs), is proposed in the paper. A standard sensor network consists of a set of sensors, which are small electronic devices capable of collecting data on certain phenomena from a defined area and then to process and transmit them, as well as a set of sinks (hubs, gates) intended for gathering data and providing them to users (Fig. 1). The main functionalities of sensors concern gathering of data from measurement points located in a monitored area, collecting these data as well as transmitting them directly to a sink or to another sensor. So, after measuring of data, the sensor can transmit them to the final destination or to serve as a broker of data between a measurement point and a sink. Sensors usually work in an environment that makes their constant maintenance impossible, so the proper energy management is the crucial task enabling the maximization of the total execution time of WSNs. It does not concern sinks which, unlike sensors, are constantly supplied facilities capable to collect and store measured data. All sensors and sinks, which constitute WSN, are wirelessly connected. The development of WSNs enables the significant extension of their applications beyond the original military usage. It is possible by advances in miniaturized mechatronic systems, and first of all, in a wireless communication. So, contemporary researches on WSNs have interdisciplinary nature and belong not only to computer science but also to metrology, electrical engineering and telecommunication. In general, they consist in: designing of sensors, their location (deployment) in an appropriate working environment, determination of coverage areas for sensors, collection and transmission of data, minimization of the energy consumption by sensors, ensuring the security of data during measurement, guarantee of the quality of services, solving various particular issues connected with mobility of sensors, e.g. [17].

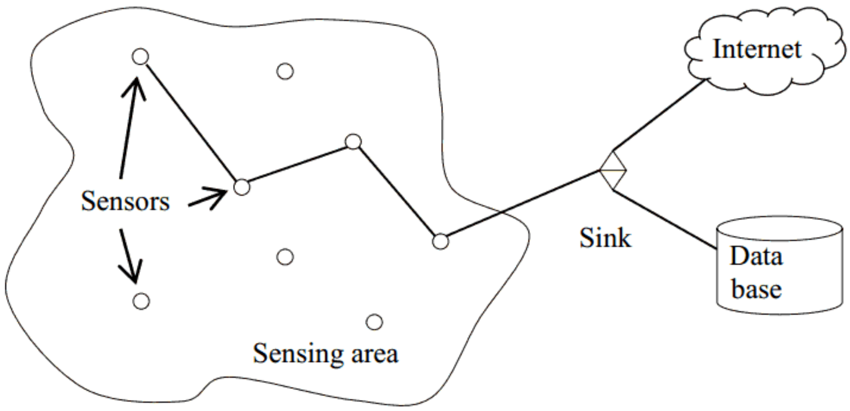

Figure 1. General structure of sensor network.

The development of both stationary and mobile WSNs also generates interesting problems in the area of operations research, which need solutions. Management of WSNs, which leads to the extension of the execution time of sensors, is the main challenge. The most important research problems from this scope for stationary networks are: location of facilities, routing of data acquired by sensors to sinks possibly via other sensors treated as brokers in the transmission as well as coverage of sensing areas by sensors, e.g. [4] and [6]. These problems are solved separately in many works such as [26], [25]. The coverage problem is particularly important. The greater range of sensors requires the increase of energy consumption which is adverse from the optimization of execution time or, equivalently, of energy consumption point of view. On the other hand, the application of more efficient facilities is more expensive and, in a consequence, increases the total cost of WSN. In the literature, many different methods to formulate and solve the coverage problem are reported, e.g.: the binary formulation ([26]), the probabilistic approach ([10]). A special case of the problem is also considered when sensors have to operate a finite number of measurement points rather than a planar area with an infinite number of points. Then, the coverage problem can be expressed with the use of values of corresponding variables characterizing the problem, e.g. number of units of data that need to be acquired by sensors. Such an approach is used in this work. It is assumed that the coverage depends both on the locations of sensors and on the cost of their purchase, due to the fact that more expensive sensors have a greater sensing range. Routing problems have been comprehensively investigated for computer networks. However, the corresponding results need the adaptation for WSNs. In order to minimize the energy, the multihop transmission is often used in wireless sensor networks. It means that sensors serve as brokers in the transmission and just forward data to other sensors.

The location of sinks and sensors is obviously connected with the data transmission if costs of their purchase and distribution are additionally taken into account. Thus, it is strongly recommended to consider the combination of these problems. The location problem in WSNs is often investigated together with the coverage problem, e.g. [23], [27]. In some works, it is formulated and solved as the clustering problem ([1]). All three problems, i.e.: the location of sinks, the routing of data and the coverage of sensing area by sensors are investigated jointly in [10] where the optimization criterion expresses the minimization of energy required to transmit data among sensors and sinks, with constraints imposed on the maximum cost of network deployment and on the minimum level of area covered by each sensor. In this paper, the different concept of the criterion is assumed which expresses the total cost of collection and transmission of data as well as of location of sensors and sinks. It is also assumed that the cost of transmission is directly proportional to the energy consumption. The classical production-transportation problem with the replacement of transport requirements by routing ones has been referred to.

The remaining text is organized as follows. Formulation of the problem for single commodity data together with its analysis is given in Section 2. Section 3 presents the heuristic solution algorithm which uses solutions of the well-known circulation problem in directed graphs. Results of numerical experiments assessing the algorithm are discussed in Section 4. Final remarks complete the paper.

2. Problem formulation and analysis

Let us consider WSN consisting of S potential sensor locations which form a set $\mathbf{S} = \{1, 2, ..., i, ..., S\}$. The aim of the sensors is to acquire data from P measurement points constituting a set $\mathbf{P} = \{S+1, S+2, ..., i, ..., S+P\}$. WSN considered includes also U potential sink locations which form a set $\mathbf{U} = \{S+P+1, S+P+2, ..., i, ..., S+P+U\}$. The following variables are defined to describe WSN:

c_{ij} - unit cost of data collection by jth sensor form ith measurement point for $j \in \mathbf{S}$ and $i \in \mathbf{P}$ or unit cost of data transmission between sensors i and j or form ith sensor to jth sink for $i, j \in \mathbf{S}$ or $i \in \mathbf{S}, j \in \mathbf{U}$, respectively; it is assumed that $c_{ii} = +\infty, i \in \mathbf{S}$,

s_i – location cost of ith sensor, $i \in \mathbf{S}$ (cost of purchase and deployment),

u_i – location cost of ith sink, $i \in \mathbf{U}$ (cost of purchase and deployment),

d_i - number of data units which have to be collected from ith measurement point, $i \in \mathbf{P}$,

$d = \sum_{k \in \mathbf{P}} d_k$ - number of all data units to be measured and transmitted.

Let us introduce the following decision variables:

$x_{ij} \in \{0, 1, 2, ..., d\}$ - number of data units collected by jth sensor from ith measurement point for $j \in \mathbf{S}$, $i \in \mathbf{P}$ or number of data units transmitted between sensors i and j or from ith sensor to jth sink for $i, j \in \mathbf{S}$ or $i \in \mathbf{S}, j \in \mathbf{U}$, respectively; it is assumed that $x_{ii} = 0, c_{ii}x_{ii} = +\infty, i \in \mathbf{S}$,

y_i – deployment of a sensor at ith prospective location, $y_i = 1(0)$ – sensor is deployed at ith location (otherwise), $i \in \mathbf{S}$,

v_i – deployment of a sink at ith prospective location, $v_i = 1(0)$ – sink is deployed at ith location i (otherwise), $i \in \mathbf{U}$.

The following constraints enable us to determine feasible solutions $\mathbf{x} = \{x_{ij} : i \in \mathbf{P}, j \in$

$\mathbf{S} \vee i,j \in \mathbf{S} \vee i \in \mathbf{S}, j \in \mathbf{U}\}$, $\mathbf{y} = \{y_i : i \in \mathbf{S}\}$ and $\mathbf{v} = \{v_i : i \in \mathbf{U}\}$ being respectively responsible for data collection or transmission, location of sensors and locations of sinks:

$$\sum_{j \in \mathbf{S}} x_{ij} = d_i, i \in \mathbf{P}, \tag{1}$$

$$\sum_{j \in \mathbf{S} \cup \mathbf{U}} x_{ij} = \sum_{j \in \mathbf{S} \cup \mathbf{P}} x_{ji}, i \in \mathbf{S}, \tag{2}$$

$$\sum_{j \in \mathbf{S} \cup \mathbf{U}} x_{ij} \leq d y_i, i \in \mathbf{S}, \tag{3}$$

$$\sum_{j \in \mathbf{S}} x_{ji} \leq d v_i, i \in \mathbf{U}, \tag{4}$$

$$x_{ij} \in \{0, 1, ..., d\}, i \in \mathbf{P}, j \in \mathbf{S} \vee i,j \in \mathbf{S} \vee i \in \mathbf{S}, j \in \mathbf{U}, \tag{5}$$

$$y_i \in \{0, 1\}, i \in \mathbf{S}, \tag{6}$$

$$v_i \in \{0, 1\}, i \in \mathbf{U}. \tag{7}$$

Equations (1) ensure that required number of data units is collected from each measurement point. Constraints (2) make it possible to balance between input and output data units for each sensor. Two subsequent constraints guarantee that no more than all collected data units can be transmitted from each sensor and received by each sink, respectively. In a consequence, the collected data units can be transmitted only among deployed sensors and sinks. The remaining requirements provide the domains for the decision variables. The joint problem of location, coverage and routing, referred to as LCRP, consists in the determination of decisions $\mathbf{x}, \mathbf{y}$ and $\mathbf{v}$, feasible with respect to (1)–(7), minimizing the total cost Q of data collection and transmission as well as sensors and sinks locations which is the sum of individual costs, i.e.

$$Q(\mathbf{x}, \mathbf{y}, \mathbf{v}) = \sum_{i \in \mathbf{S}} s_i y_i + \sum_{i \in \mathbf{U}} u_i v_i + \sum_{i \in \mathbf{S}} \sum_{j \in \mathbf{S} \cup \mathbf{U}} c_{ij} x_{ij} + \sum_{i \in \mathbf{P}} \sum_{j \in \mathbf{S}} c_{ij} x_{ij}. \tag{8}$$

Sets $\mathbf{P}$, $\mathbf{S}$, $\mathbf{U}$, all unit costs $\mathbf{c} = \{c_{ij} : i \in \mathbf{P}, j \in \mathbf{S} \vee i,j \in \mathbf{S} \vee i \in \mathbf{S}, j \in \mathbf{U}\}$, number of data units $\mathbf{d} = \{d_i : i \in \mathbf{P}\}$ as well as location costs $\mathbf{s} = \{s_i : i \in \mathbf{S}\}$, $\mathbf{u} = \{u_i : i \in \mathbf{U}\}$ are given. Moreover, the single commodity case is considered which means the homogeneity of data units. Without loss of generality, the case is considered when only one type of sensors can be deployed at a single location. Considering different types of sensors at a single location would require the deployment of many sensors with different costs c_{ij}, s_i and would result only in the greater size of problem. According to the theorem, LCRP is NP-hard. In the proof, it is justified that the NP-hard uncapcitated facility location problem is a special case of LCRP.

Theorem. LCRP is NP-hard.

Proof. Let us consider a special case of LCRP when $d_i = 1, i \in \mathbf{P}$; $u_i = 0, i \in \mathbf{U}$; $c_{ij} = 0, i,j \in \mathbf{S} \vee i \in \mathbf{S}, j \in \mathbf{U}$. It is easy to see that criterion (8) takes then the form:

$$\bar{Q}(\mathbf{x}, \mathbf{y}) = \sum_{i \in \mathbf{S}} s_i y_i + \sum_{i \in \mathbf{P}} \sum_{j \in \mathbf{S}} c_{ij} x_{ij}. \tag{9}$$

Constraints (2), (4) and (7) are inactive due to the costless deployment of sinks and transmissions to them. In particular, the left hand side of (2) can have any value. Conditions (1), (5) and (6) remain unchanged. Inequalities (3) are transformed. Namely, let us notice that for the assumptions accepted, there are many optimal solutions with different transfers of data units among sensors (each data unit can be passed directly to a sink or via sequence of deployed sensors) and with the same optimal zero cost. After replacing the left-hand side of (3) by the right-hand side of (2), we get:

$$\sum_{j\in\mathbf{P}} x_{ji} + \sum_{j\in\mathbf{S}} x_{ji} \le P y_i, i \in \mathbf{S}, \tag{10}$$

and consequently

$$\sum_{j\in\mathbf{P}} x_{ji} \le P y_i, i \in \mathbf{S}. \tag{11}$$

According to (1) and the assumption $d_i = 1, i \in \mathbf{P}$, no more than P data units can be collected by each sensor, i.e. $(\sum_{j\in\mathbf{P}} x_{ji})/P \le 1, i \in \mathbf{S}$. So, we can transform (11) to the equivalent form:

$$x_{ji} \le y_i, i \in \mathbf{S}, j \in \mathbf{P} \tag{12}$$

which replaces (3) for the special case. Finally, problem (9) with constraints (1), (5), (6), (12) is the NP-hard uncapacitated facility location problem ([5]).

□

3. Solution algorithm

The main idea of the algorithm proposed consists in the application of solution of the suitably defined circulation problem, e.g. [12]. Such problem, for a directed graph $\mathbf{G} = (\mathbf{V}, \mathbf{E})$ with the set of vertices $\mathbf{V}$ and the set of arcs $\mathbf{E}$ as well as for given minimal $\underline{f}_e, e \in \mathbf{E}$ and maximal $\bar{f}_e, e \in \mathbf{E}$ values of arc flows and unit costs of flows through arcs $c_e, e \in \mathbf{E}$, deals with the determination of the arc flows $f_e, e \in \mathbf{E}$ to minimize the total flow cost in the graph, i.e.:

$$min \sum_{e\in\mathbf{E}} c_e f_e, \tag{13}$$

subject to

$$\underline{f}_e \le f_e \le \bar{f}_e, e \in \mathbf{E}, \tag{14}$$

$$\sum_{e\in\delta^+(v)} f_e = \sum_{e\in\delta^-(v)} f_e \tag{15}$$

where $\delta^+(v)$ and $\delta^-(v)$ are a set of incoming arcs and a set of outgoing arcs for vertex v, respectively. Minimization (13) together with constraints (14) and (15) constitute easily solvable linear programming problem. Moreover, we can get the integer solution suitable for WSN according to the property known as the Hoffmans theorem ([12]), which ensures having the integer solutions for the integer flow constraints $\underline{f}_e$ and $\bar{f}_e$. So, let us define the corresponding circulation problem. The set of vertices $\mathbf{V}$ is the sum of disjoint sets $\mathbf{P}$, $\mathbf{S}$, $\mathbf{U}$ and theirs copies $\mathbf{P}'$, $\mathbf{S}'$, $\mathbf{U}'$, i.e. $V = \{\mathbf{P} \cup \mathbf{S} \cup \mathbf{U} \cup \mathbf{P}' \cup \mathbf{S}' \cup \mathbf{U}'\}$. Inserting the copies of vertices into the graph allow us to take into account the location costs in the circulation problem as

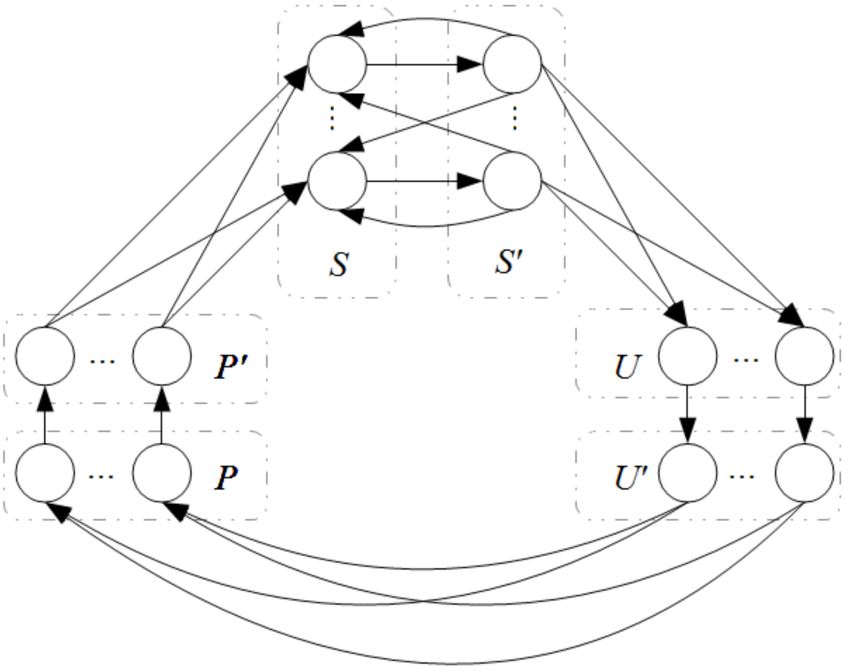

Figure 2. Circulation graph scheme

well as to represent the data collection by sensors from the measurement points. The set of arcs $\mathbf{E}$ comprises the following connections between the pairs of vertices from $\mathbf{V}$ among which one can distinguish: the connections between the vertices from sets $\mathbf{P}, \mathbf{S}, \mathbf{U}$ and their counterparts from sets $\mathbf{P}', \mathbf{S}', \mathbf{U}'$, and the connections between all pairs of vertices from sets $\mathbf{A}$ and $\mathbf{B}$, namely (notation $\{\mathbf{A} \rightarrow \mathbf{B}\}$ describes connections between vertices from sets $\mathbf{A}$ and $\mathbf{B}$, where $\mathbf{A}, \mathbf{B} \in \{\mathbf{P}, \mathbf{P}', \mathbf{S}, \mathbf{S}', \mathbf{U}, \mathbf{U}'\}$):

a) $\{\mathbf{P} \rightarrow \mathbf{P}'\}$, $\{\mathbf{S} \rightarrow \mathbf{S}'\}$, $\{\mathbf{U} \rightarrow \mathbf{U}'\}$ – sets of arcs connecting elements of $\mathbf{P}, \mathbf{S}, \mathbf{U}$ only with their counterparts from $\mathbf{P}', \mathbf{S}', \mathbf{U}'$, respectively,

b) $\{\mathbf{P}' \rightarrow \mathbf{S}\}$, $\{\mathbf{S}' \rightarrow \mathbf{S}\}$, $\{\mathbf{S}' \rightarrow \mathbf{U}\}$, $\{\mathbf{U}' \rightarrow \mathbf{P}\}$ – sets of arcs connecting all elements of $\mathbf{P}', \mathbf{S}', \mathbf{S}', \mathbf{U}'$ with all elements of $\mathbf{S}, \mathbf{S}, \mathbf{U}, \mathbf{P}$, respectively.

The defined graph $\mathbf{G} = (\mathbf{V}, \mathbf{E})$ is shown in Fig. 2. Its interpretation and relation to the problem investigated are as follows. The arcs between sets of original elements ($\mathbf{P}$ $\mathbf{S}$, $\mathbf{U}$) and their copies allow taking into account the cost of sensors ($\{\mathbf{S} \rightarrow \mathbf{S}'\}$) and sinks ($\{\mathbf{U} \rightarrow \mathbf{U}'\}$), as well as the collection of required number of data units from each measurement data point ($\{\mathbf{P} \rightarrow \mathbf{P}'\}$). Other connections represent data transmissions between: measurement points and sensors ($\{\mathbf{P}' \rightarrow \mathbf{S}\}$), sensors ($\{\mathbf{S}' \rightarrow \mathbf{S}\}$), sensors and sinks ($\{\mathbf{S}' \rightarrow \mathbf{U}\}$). Remaining connections $\{\mathbf{U}' \rightarrow \mathbf{P}\}$ have been introduced artificially in order to ensure the circulation in the graph. The arc flows of these connections have not any interpretation for LCRP. According to the definition of the circulation problem, any element of the graph cannot generate or collect data.

The values of all arc flows f_e in the circulation problem are optimization variables, so they are not known before solving the problem. Thus, the edge flows of $\{\mathbf{S} \rightarrow \mathbf{S}'\}$ and $\{\mathbf{U} \rightarrow \mathbf{U}'\}$ cannot coincide with the actual location costs s_i and u_i , respectively. In fact, these costs are results generated by the solution algorithm, and the optimal solution would be achieved only if the corresponding flow values were equal to the location costs known a priori. Generally, it is not true, and, in a consequence, a heuristic solution algorithm is proposed. In order to ensure the integral solution, constraints are defined as follows:

$$\underline{f}_e = \begin{cases} d_i, & e \in \{\mathbf{P} \to \mathbf{P}'\}, \\ 0, & otherwise, \end{cases} \quad \bar{f}_e = \begin{cases} d_i, & e \in \{\mathbf{P} \to \mathbf{P}'\}, \\ +\infty, & otherwise \end{cases} \tag{16}$$

where $e \triangleq (i, j)$ is the arc between ith element from set $\mathbf{P}$ and the corresponding one from set $\mathbf{P}'$. The unit costs assigned to each arc are defined as:

$$c_e = \begin{cases} 0, & e \in \{\mathbf{P} \to \mathbf{P}'\} \cup \{\mathbf{U}' \to \mathbf{P}\}, \\ c_{ij}, & e \in \{\mathbf{P}' \to \mathbf{S}\} \cup \{\mathbf{S}' \to \mathbf{S}\} \cup \{\mathbf{S}' \to \mathbf{U}\}, \\ \frac{s_i}{\tilde{s}_i}, & e \in \{\mathbf{S} \to \mathbf{S}'\}, \\ \frac{u_i}{\tilde{u}_i}, & e \in \{\mathbf{U} \to \mathbf{U}'\} \end{cases} \tag{17}$$

where c_{ij}, s_i, u_i are given, whereas $\tilde{s}_i$, $\tilde{u}_i$ undergo setting during the execution of the algorithm to make values of c_e as close as possible to the corresponding location costs of LCRP.

The circulation problem defined in this way is similar to LCRP. The values of decision variables x_{ij} are equal to flows f_e for $e \in \{\mathbf{P}' \to \mathbf{S}\} \cup \{\mathbf{S}' \to \mathbf{S}\} \cup \{\mathbf{S}' \to \mathbf{U}\}$ while the values of the remaining decision variables can be calculated using the following formulas: $y_i = sgn(f_e), e \in \{\mathbf{S} \to \mathbf{S}'\}$, and $v_i = sgn(f_e), e \in \{\mathbf{U} \to \mathbf{U}'\}$. The circulation cost is the sum:

$$\sum_{e \in \{\mathbf{S} \to \mathbf{S}'\}} c_e f_e + \sum_{e \in \{\mathbf{U} \to \mathbf{U}'\}} c_e f_e + \sum_{e \in \{\mathbf{S}' \to \mathbf{S}\} \cup \{\mathbf{P}' \to \mathbf{S}\} \cup \{\mathbf{S}' \to \mathbf{U}\}} c_e f_e \tag{18}$$

where corresponding parts are: location costs of sensors and sinks as well as data collection and transmission costs. Unfortunately, the values of two former ones are not equal to the location costs expressed by two first elements of sum (8). The aim of the algorithm is to approach the values of (18) and (8) as close as possible. Constraint (1) follows directly from (17) whereas conditions (2)–(4) from the flow condition (15).

The essence of the algorithm consists in changing of the arc unit costs $\tilde{s}_i$, $\tilde{u}_i$ referred to as $\tilde{s}_i(n)$, $\tilde{u}_i(n)$ in each iteration by means of the update procedure. For given: graph $\mathbf{G} = (\mathbf{V}, \mathbf{E})$, the form of unit collection/transmission costs c_e given as (17), initial values of location costs $\tilde{s}_i(0) = \tilde{u}_i(0) = d$, η as the parameter of the stop condition and $n = 0$, the algorithm is composed of three following steps:

1. Solve the circulation problem to achieve arc flows $f_e(n), e \in \mathbf{E}$.
2. If stop condition, understood as the lack of the improvement of solution in last η iterations, is not fulfilled, go to Step 3. Otherwise, return solution: $x_{ij} = f_e(n), e \in \{\mathbf{P}' \to \mathbf{S}\} \cup \{\mathbf{S}' \to \mathbf{S}\} \cup \{\mathbf{S}' \to \mathbf{U}\}$, $y_i = sgn(f_e), e \in \{\mathbf{S} \to \mathbf{S}'\}$, $v_i = sgn(f_e), e \in \{\mathbf{U} \to \mathbf{U}'\}$ and stop the algorithm.
3. Update location costs using procedures θ_s and θ_u, i.e. $\tilde{s}_i(n+1) = \theta_s(\tilde{s}_i(n), f_e(n))$, $\tilde{u}_i(n+1) = \theta_s(\tilde{u}_i(n), f_e(n))$ for $f_e(n) > 0$. Otherwise, i.e. when $f_e(n) \leq 0$, substitute $\tilde{s}_i(n+1) = \tilde{s}_i(n)$, $\tilde{u}_i(n+1) = \tilde{u}_i(n)$. Then, set $n = n+1$ and go to Step 1.

Two update procedures are suggested: $\tilde{x}_i(n+1) = f_e(n)$ and $\tilde{x}_i(n+1) = (\tilde{x}_i(n) + f_e(n))/2$ where $\tilde{x}_i(\cdot) \in \{\tilde{s}_i, \tilde{u}_i\}, e \triangleq (i, j)$ and j is the counterpart of i. The former one enable us to set the values of current location costs for sensors and sinks as s_i and u_i, respectively. Please note that $c_e f_e = f_e \frac{s_i}{f_e} = s_i$ and $c_e f_e = f_e \frac{u_i}{f_e} = u_i$. The latter procedure takes into account not only the values of actual location costs s_i and c_i but also the corresponding values calculated in the previous iteration of the algorithm. The simple average is proposed to combine both values. The numerical experiments showed that the application of the first update procedure leads to the similar results in shorter time.

4. Numerical Experiments

In order to determine the quality of the heuristic solution algorithm proposed, the preliminary numerical experiments were performed. The values of the problem data were randomly generated within the following intervals according to the uniform distribution:

a. number of data units d_i collected at measurement points from discrete set $\{1, 2, ..., 10\}$,
b. unit collection/transmission costs c_{ij} proportionally to the distance between pairs of points i and j which coordinates were randomly generated within a X side square,
c. location costs s_i and u_i from intervals which bounds depend on c_{ij}, i.e. s_i is randomly generated from interval $[\bar{c}_i, m\bar{c}_i]$ where $m = 2, 3, ...$, $\bar{c}_i = (\sum_{j\in\mathbf{P}\cup\mathbf{S}\cup\mathbf{U}} c_{ij})/(P + S + U)$, while $u_i = 10s_i$.

Two configurations of WSN were tested where mutual proportion among the number of measurement points P, sensors S and sinks U were constant. For the 1st and the 2nd configuration, proportions $1 : 1 : 1$, i.e. $P = S = U = k$ and $17 : 3 : 1$, i.e. $P = 17k$, $S = 3k$, $U = k$ were kept, respectively where $k = 1, 2, ...$ is a parameter of the numerical experiments. Furthermore, it was assumed that $\eta = 25$, $X = 1000$ and $\tilde{x}_i(n + 1) = f_e(n), \tilde{x}_i(\cdot) \in \{\tilde{s}_i, \tilde{u}_i\}$.

The solutions generated by the heuristic algorithm were evaluated with the reference to the optimal solutions obtained by solver GLPK (*http://www.gnu.org/s/glpk*) using the performance indices $\epsilon_Q = Q/Q^*$ and $\epsilon_T = T^*/T$ where Q, T and Q^*, T^* are the total cost, the execution time of the algorithm for the heuristic and the optimal algorithm, respectively.

The quality of results generated by the heuristic algorithm was checked for different values of parameters m and k. The examples of results for both configurations are presented in Tables 1 and 2 where corresponding values are averages of 5 independent runs of the algorithm.

Table 1. Dependence of ϵ_Q and ϵ_T on k and m for the 1st configuration

	ϵ_Q				ϵ_T			
$k \setminus m$	10	50	250	1250	10	50	250	1250
28	1,19	1,15	1,26	1,55	9,2	1,0	1,0	1,0
35	1,26	1,22	1,17	1,39	9,4	2,0	1,0	1,0
42	1,20	1,17	1,40	1,47	34,0	2,8	1,2	1,0
49	1,23	1,42	1,22	1,50	59,8	5,4	1,0	1,0
56	1,30	1,28	1,29	1,45	308,0	4,0	1,6	1,0
63	1,30	1,37	1,31	1,29	493,2	14,2	1,0	1,0

The following main conclusions result from the experiments conducted:

a. The heuristic algorithm gives worse results for greater values of parameter m, i.e., for the big difference between the location costs of sensors and sinks and the average unit cost of data collection/transmission.
b. For increasing values of m, the optimal solutions are found quicker, in spite of the constant number of constraints and optimization variables for the integer linear programming problem.

Table 2. Dependence of ϵ_Q and ϵ_T on k and m for the 2nd configuration

	ϵ_Q				ϵ_T			
$k \backslash m$	10	50	250	1250	10	50	250	1250
14	1,10	1,18	1,20	1,12	23,0	6,0	1,0	1,0
15	1,08	1,14	1,20	1,12	41,2	6,8	1,0	1,0
16	1,10	1,17	1,19	1,30	53,6	6,6	1,4	1,0
17	1,10	1,17	1,16	1,22	81,8	13,2	1,8	1,0
18	1,10	1,17	1,23	1,24	69,4	14,0	1,2	1,0

c. For small values of m unlike the large values of this parameter, the heuristic algorithm performs better for both the total cost and the execution time.
d. The heuristic algorithm performs better for the 2nd configuration (which is closer to the real-world applications) when the number of measurement points is much bigger than the number of sensors and/or sinks. It is worth noting that the quality of results for small m is insensitive on the increase of values of k. This means that when the location costs of sensors and sinks are not significantly greater than the average cost of data unit collection/transmission, this algorithm generates solutions of the similar quality (i.e. 20% worse than the optimal ones, as the maximum) for increasing sizes of problem instances.

In the experiments reported, the number of WSN elements $P+S+U$ is ranged from 84 to 189 and from 294 to 378 for the 1st and 2nd configuration, respectively. For the larger problem instances, the application of the solver is impossible due to the high memory complexity of the problem.

5. Final remarks

The paper considers the selected version of the joint location, coverage and routing problem for wireless sensor networks. The heuristic solution algorithm has been proposed which crucial part consists in solving the known circulation problem in a directed graph. The initial numerical experiments assessing the quality of the algorithm have been also conducted. They have shown that for the instances tested it is possible to obtain solutions at most 1.55 times worse than the optimal ones.

During further works which are planned, we will be concerned with the following research directions.

a. Improvement of the heuristic algorithm will be continued. Preliminary investigations confirmed that the application of other updating procedures is promising.
b. Consideration of new versions of the problem, e.g. with limitations on the data units transmission and on the capacity of sensors as well as taking into account time restrictions on the data units collections (the version with time windows).
c. Investigation of related problems. The following ones seem to be the most important: the problem with a new criterion expressing only the energy consumption during all activities in WSNs and with investments costs as constraints; the extension of researches on mobile wireless sensor networks; corresponding problems for various types of data collected (multicommodity cases).

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