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The mean error estimation of TOPSIS method using a fuzzy reference models

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Abstract: *The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a commonly used multi-criteria decision-making method. A number of authors have proposed improvements, known as extensions, of the TOPSIS method, but these extensions have not been examined with respect to accuracy. Accuracy estimation is very difficult because reference values for the obtained results are not known, therefore, the results of each extension are compared to one another. In this paper, the author propose a new method to estimate the mean error of TOPSIS with the use of a fuzzy reference model (FRM). This method provides reference values. In experiments involving 1,000 models, 28 million cases are simulated to estimate the mean error. Results of four commonly used normalization procedures were compared. Additionally, the author demonstrated the relationship between the value of the mean error and the nonlinearity of models and a number of alternatives.*

Keywords: *TOPSIS, accuracy, mean error, fuzzy logic, decision-making*

1. Introduction

Multi-criteria decision-making (MCDM) techniques are important and popular mathematical methods used in a variety of human activities. Generally, a decision making process involves finding the best option of all the feasible alternatives. Usually, this is achieved by calculating the preference value of each alternative [1].

One of the most frequently used MCDM methods is the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). This technique was first proposed by Hwang and Yoon [2] and has been applied in a number of fields, including energy [3, 4, 5, 6, 7], medicine [8, 9, 10, 11], engineering and manufacturing systems [12, 13, 14, 15, 16, 17], safety and environmental fields [18, 19, 20, 21, 22], chemical engineering [5, 23, 24], and water resources studies [5, 20, 23, 25]. The classical TOPSIS method has been extended to facilitate decision-making in a fuzzy environment. This is a very important and popular extension [3, 4, 6, 8, 12, 13, 14, 15, 16, 18, 19, 20, 25, 26, 27, 28, 29]. The use of interval numbers is also an important improvement [30, 31, 32, 33]. For example, the fuzzy extension of TOPSIS has been used to express the probability of success for pancreatic islet transplantation [8].

There are four major variants of currently used normalization methods for TOPSIS [34, 35]. The final TOPSIS results vary depending on the method of normalization, affecting the rank of alternatives. This phenomenon is called Rank Reversals [36]. Milani et al. investigated the effect of normalization norms on TOPSIS [34, 37, 38]. They concluded that different norms introduce different relative closeness of attributes, yet for linear norms, this is not sufficient to change the rank of preferred alternatives.

In this paper, the author assessed the mean error of normalization procedures for TOPSIS using a fuzzy reference model (FRM) and a procedure developed by the author.

2. TOPSIS procedure

The classical TOPSIS method is based on the idea that the best alternative should have the shortest geometric distance from the positive ideal solution (PIS) and the longest distance from the negative ideal solution (NIS) [2, 39, 40]. PIS and NIS are easiest to identify when all of the criteria are monotonic (either increasing/profit attributes or decreasing/cost attributes). This is a common assumption when using TOPSIS.

For example, assume that a decision-making problem is based on m alternatives, A_1, A_2, A_3, ..., A_m, and n monotonic criteria, C_1, C_2, C_3, ..., C_n. Using these data, an original score of the decision matrix $D[x_{ij}]_{mxn}$ is created, where x_{ij} is the evaluation of alternative A_i for criterion C_j. Finally, $W = (w_1, w_2, w_3, ..., w_n)$ is the vector of criteria weights, where $\sum_{j=1}^{n} w_j = 1$. With the above assumptions, TOPSIS is performed using the following six steps:

Step 1: Create an evaluation matrix $D[x_{ij}]_{mxn}$ consisting of m alternatives and n criteria.
Step 2: Normalize the evaluation matrix $D[x_{ij}]_{mxn}$ using one of the normalization methods [34, 35]:

Method $\#1$ is linear and defined for profit attributes by equation (1) and for cost attributes by equation (2):

$$r_{ij} = \frac{x_{ij} - min_i(x_{ij})}{max_i(x_{ij}) - min_i(x_{ij})} \tag{1}$$

$$r_{ij} = \frac{max_i(x_{ij}) - x_{ij}}{max_i(x_{ij}) - min_i(x_{ij})} \tag{2}$$

Method $\#2$ is linear and defined for profit attributes by equation (3) and for cost attributes by equation (4):

$$r_{ij} = \frac{x_{ij}}{max_i(x_{ij})} \tag{3}$$

$$r_{ij} = 1 - \frac{x_{ij}}{max_i(x_{ij})} \tag{4}$$

Method $\#3$ is linear and defined for profit attributes by equation (5) and for cost attributes by equation (6):

$$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m}(x_{ij})} \tag{5}$$

$$r_{ij} = 1 - \frac{x_{ij}}{\sum_{i=1}^{m}(x_{ij})} \tag{6}$$

Method $\#4$ is nonlinear and defined for profit attributes by equation (7) and for cost attributes by equation (8):

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m}(x_{ij})}} \tag{7}$$

$$r_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m}(x_{ij})}} \tag{8}$$

Step 3: Calculate the weighted normalized decision matrix defined by equation (9) for each element in the matrix:

$$v_{ij} = w_j \cdot r_{ij}\,, \quad i = 1, 2, ..., m; \;\; j = 1, 2, ..., n. \tag{9}$$

Step 4: Determine the PIS using equation (10) and the NIS using equation (11):

$$V_j^+ = \{v_1^+, v_2^+, v_3^+, \ldots, v_n^+\} = \{(max_i(v_{ij})|j \in K_b)(min_i(v_{ij})|j \in K_c)\} \tag{10}$$

$$V_j^+ = \{v_1^-, v_2^-, v_3^-, \ldots, v_n^-\} = \{(min_i(v_{ij})|j \in K_b)(max_i(v_{ij})|j \in K_c)\} \tag{11}$$

where K_b is a set of benefit criteria and K_c is a set of cost criteria.
Step 5: Calculate the Euclidean distance between the following:
the $i - th$ alternative (where $i = 1, 2, ..., m$) and the NIS alternative (12):

$$D_i^- = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^-)^2} \tag{12}$$

the $i - th$ alternative (where $i = 1, 2, ..., m$) and the PIS alternative (13):

$$D_i^+ = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^+)^2} \tag{13}$$

Step 6: Calculate the relative closeness to the ideal condition using equation (14):

$$C_i = \frac{D_i^-}{D_i^- + D_i^+}\,, \quad i = 1, 2, ..., m. \tag{14}$$

In this paper, the author focuses on normalization procedures using equations (1), (3), (5) and (7).

3. Proposed approach

To estimate the mean error of the normalization methods for TOPSIS, reference values are required. Proper accuracy estimation is essential for any MCDM method, but it is unheard of in the scientific literature. In this paper, the author proposes an uncomplicated approach based on fuzzy logic. For this purpose, the author has developed idea of Fuzzy Reference Model (FRM). The FRM is a multi-criteria function of the decision maker's preferences, containing information about the preferences of each alternative. Based on these models, all values of alternative preferences are known a priori. The general scheme of the estimation procedure used for TOPSIS is presented in Figure 1. The presented procedure should be repeated multiple times for many FRMs, in order to generalize result of simulations.

To create the FRM, the following steps must be performed:

Step 1: Select a number of criteria and kind of monotonicity (profit, cost and nonmonotonic).
Step 2: Build a membership function for the each criterion with a constant domain [0, 1].

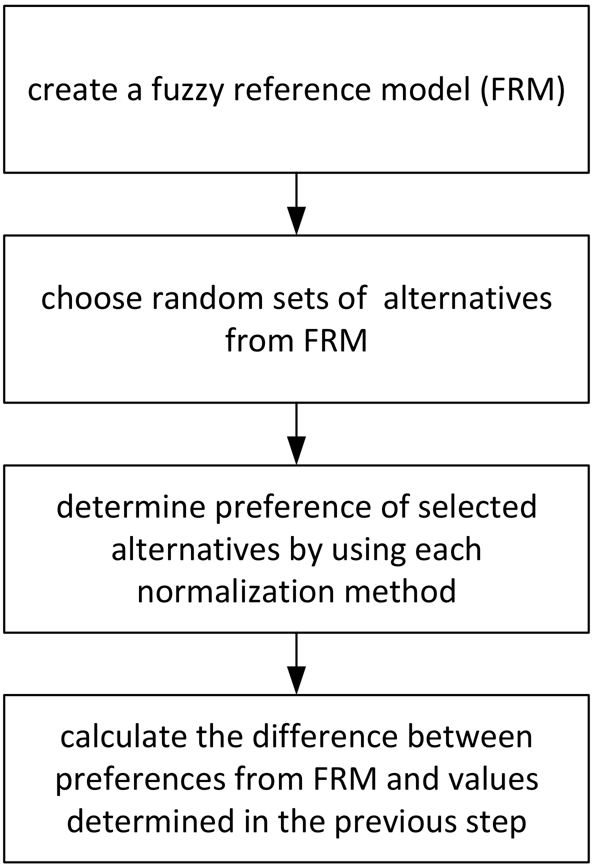

Figure 1. The proposed estimation procedure.

Step 3: Provide the evaluation value for each combination of information granules. It can be random value, but it has to respect previous assumptions.
Step 4: Create a fuzzy rule of FRM on the basis of tautology Modus Ponens [41] as follows:
$R_1 : IF(C_1 \sim a_{11})AND(C_2 \sim a_{12}) \dots (C_n \sim a_{1n})THEN(MC \sim c_1)$
$R_2 : IF(C_1 \sim a_{21})AND(C_2 \sim a_{22}) \dots (C_n \sim a_{2n})THEN(MC \sim c_2)$
$R_3 : IF(C_1 \sim a_{31})AND(C_2 \sim a_{32}) \dots (C_n \sim a_{3n})THEN(MC \sim c_3)$
. . . .
$R_r : IF(C_1 \sim a_{r1})AND(C_2 \sim a_{r2}) \dots (C_n \sim a_{rn})THEN(MC \sim c_r)$

where a_{ij} is the value of information granule for the $i-th$ rule R_i $(i = 1,2,3,...,r)$ and $j-th$ criterion C_j $(j = 1,2,3,...,n)$, MC is the multi-criteria reference model , c_i is the value of reference model for the $i-th$ rule.

In this manner, a FRM is obtained, and subsequently, a set of alternatives is randomly selected. For each alternative, the preference value from the FRM is known but the criteria are unknown. Using the least square method and reference values, the coefficients of significance criteria are obtained and then these factors are scaled to a sum equal to one. In this procedure, these numbers are the weights of the criteria. Next, the values of preference are computed for the selected alternatives using TOPSIS and the four different normalization methods. Finally, a value of error is computed using equation (15):

$$e_{ij} = |C_{iRef} - C_{ij}| \tag{15}$$

where e_{ij} is the error of the $i-th$ alternative and the $j-th$ method, C_{iRef} is the reference of the $i-th$ alternative and $j-th$ method. In this paper, many models and experiments are performed to calculate the mean value of error.

The author also investigated the dependences between the nonlinearity of a model and the value of error. For this purpose, we must calculate the level of nonlinearity of defined models. This can be accomplished by using formula [42, 43] (16):

$$N\text{-}Ind_k = \frac{\sum_{i=1}^{m} |K_i - K_{Li}|}{0.5m(K_{max} - K_{min})} \tag{16}$$

where $N\text{-}Ind_k$ is a quantitative indicator of nonlinearity, K_i is the value of objects from the FRM, K_{Li} is the value of objects from the linear model, K_{max} is the highest value of objects from the FRM, K_{min} is the lowest value of objects from the FRM and m is the number of objects.

4. Experiments and results

In this paper, the author demonstrates the proposed approach using two monotonic criteria. With two profit criteria, five-element membership functions are determined for the following information granules: 0.00, 0.25, 0.50, 0.75 and 1.00. The type of fuzzy system used in this paper applies a triangular-shaped membership function TSMF [41], as shown in Figure 2.

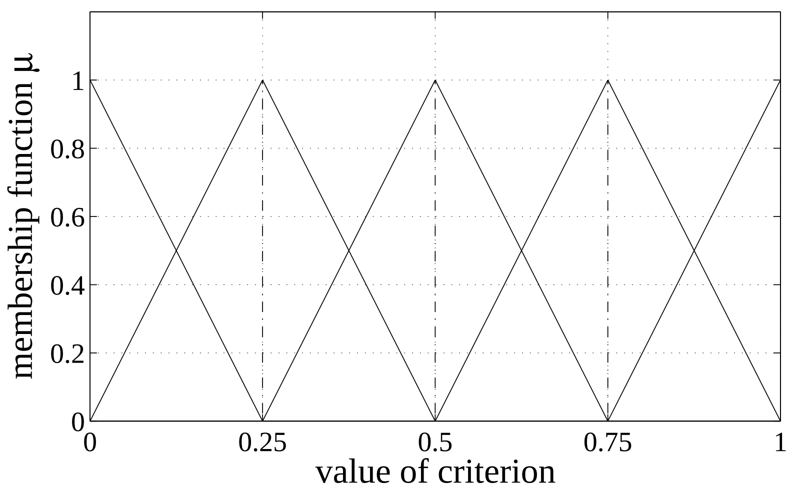

Figure 2. The triangular-shaped membership function

For each combination of information granules, the multi - criteria values are randomly selected so that the monotonicity of the criteria was preserved. An example of this action is shown in Table 1.

Based on the data from Table 1, the author identified a linear model by using the least square method to obtain significance coefficients: 0.59 for criterion 1 and 0.72 for criterion 2. After rescaling, the respective significance coefficients were 0.45 and 0.55. The example surface of the model is shown in Figure 3. The nonlinearity of this FRM model was 0.2341, as determined by using formula (16).

In this investigation, 1,000 FRMs are created. For 1000 models, the maximum error of estimation does not exeed 0.001 with statistical significance 0.05 [44, 45]. All of the models

Table 1. An example of multi-criteria values (MC) of the FRM for criterion 1 (C1) and 2 (C2).

C1 \ C2	**0.00**	**0.25**	**0.50**	**0.75**	**1.00**
0.00	0.00	0.04	0.11	0.69	0.70
0.25	0.02	0.13	0.19	0.84	0.97
0.50	0.35	0.79	0.81	0.87	0.98
0.75	0.65	0.81	0.90	0.91	0.99
1.00	0.90	0.94	0.95	0.96	1.00

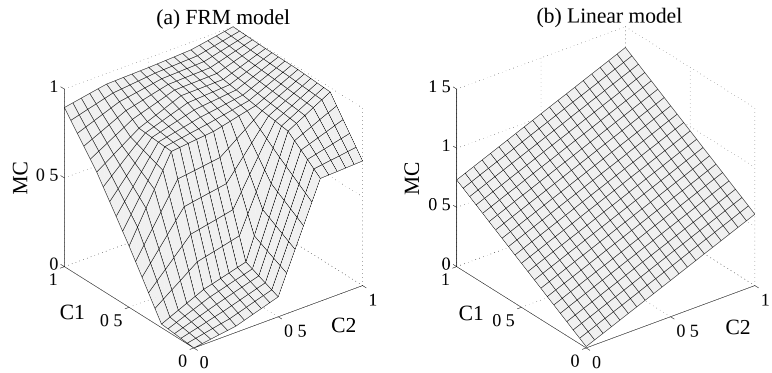

Figure 3. The example surface of FRM (a) and linear model (b)

were created in the same manner as the example described above, and the distribution of their nonlinearity is shown in Figure 4. For each created model, 28,000 sets of alternatives were

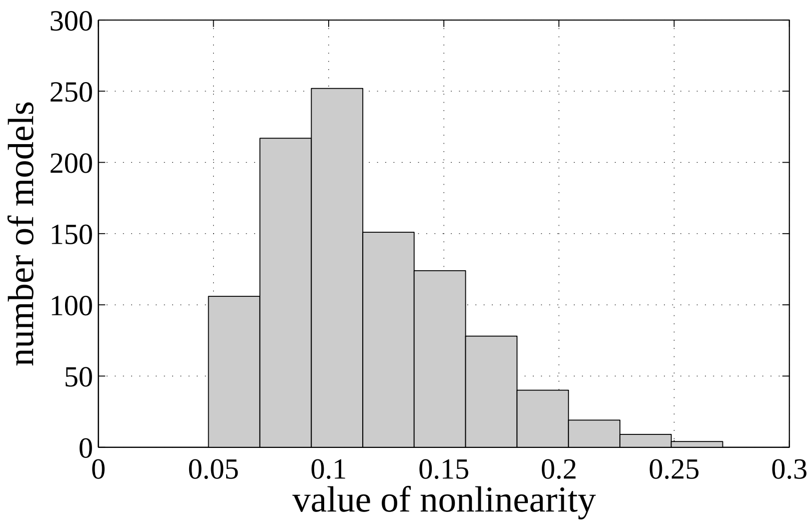

Figure 4. The distribution of nonlinearity of 1,000 generated FRMs.

randomly selected (1,000 sets of 3 alternatives, 1,000 sets of 4 alternatives, and 1,000 sets of 30 alternatives). For each set, the author used the multi-criteria value of preference from the FRM and also computed it by using TOPSIS. In this manner, the author obtained five values of preference. One was the reference value from the FRM, and four were computed from the normalization methods using equations (1), (3), (5) and (7). Finally, these values were compared with the reference values using equation (15). For any number of alternatives (in intervals from 3 to 30), 1,000,000 simulations were performed. Based on this data, analyzed normalization methods could be compared. The value of the mean error for each method is presented in Figure 5.

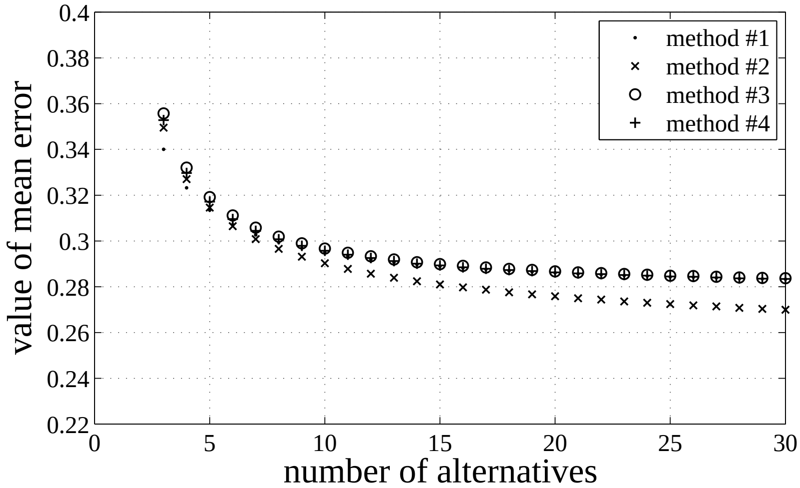

Figure 5. The comparison between analyzed normalization methods for all 1,000 models.

As shown in Figure 5, the mean error is dependent on the size of the set. Methods #1, #3 and #4 have very similar values of mean error, and method #2 has significantly higher accuracy for sets with more than 5 alternatives. Next, the relationships are examined between nonlinearity and the mean error. The 250 models with the smallest nonlinearity are selected (less than 0.0846), as shown in Figure 6. The values of the mean error shown in Figure 6 were significantly lower than the values shown in Figure 5 However, the relationship between the number of alternatives and the value of mean error remained the same. The only difference is that the method #2 has a smaller advantage than in case of all models.

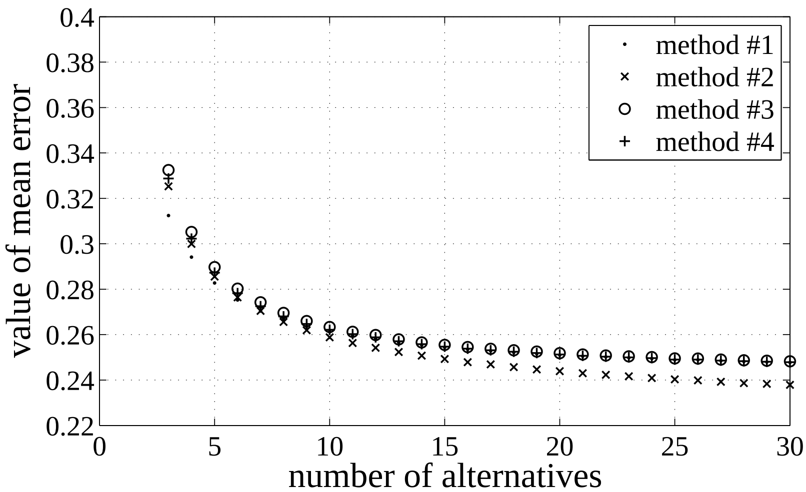

Figure 6. The comparison between analyzed normalization methods for 250 models with the smallest nonlinearity.

In the next step, author selected the 250 models with the greatest nonlinearity (greater than 0.1396), as shown in Figure 7. Method #2 offered the greatest advantage due to the relationship between the level of nonlinearity and the mean error value. In the last step, the

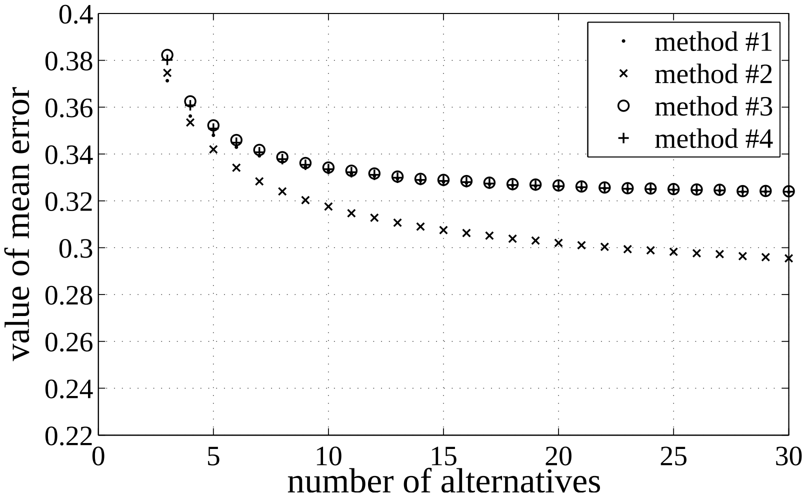

Figure 7. The comparison between analyzed normalization methods for 250 models with the greatest nonlinearity.

typical areas of variation of nonlinearity are selected (values greater than 0.0846 but less than 0.1346). In the investigation, 500 models were used. The results are presented in Figure 8. The characteristics of the graph are very similar to the plot obtained when all of the models were included (Figure 5.).

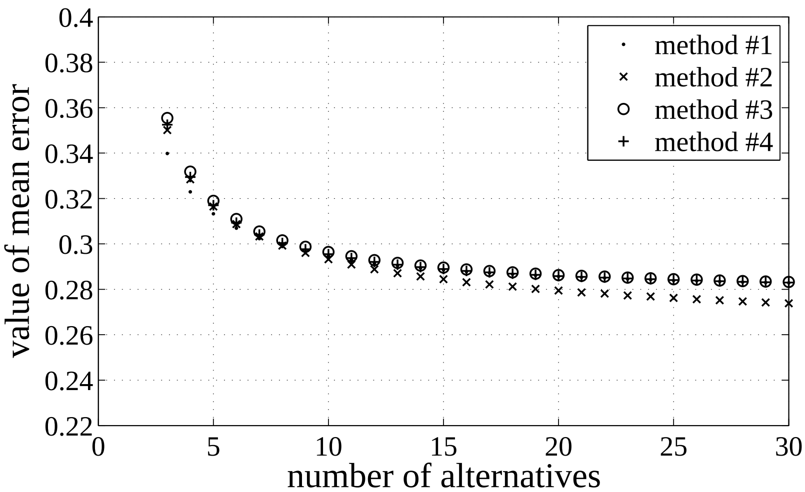

Figure 8. The comparison between analyzed normalization methods for 500 models with a typical area of variation of nonlinearity.

5. Conclusions

A new approach to estimate the accuracy of TOPSIS method with the use of FRM is proposed. In this paper, the procedure to estimate the mean error of TOPSIS method is demonstrated. This procedure was implemented in MATLAB and tested for two monotonic criteria (both contained in a set of benefit criteria). One thousand FRMs were created, and 28,000 simulations were performed for each FRM. The mean value of error was convergent and dependent on the number of alternatives. If the number of alternatives are increased, the value of mean error is reduced. Therefore, we can say that the accuracy of TOPSIS is dependent on the level of nonlinearity of the decision-making model. The mean value of error is in the range from about 0.24 to 0.38 for problem with two profit criteria, which resulted in a 24-38% relative error. Additionally, results of four normalization methods are compared. Method #1 was shown to be the best choice for a small number of alternatives (five or less); method #2 is a better choice for a larger number of alternatives.

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