Facial surface reconstruction in 3D format

Nadezhda Shchegoleva
Department of Mathematical Computer Software, Saint Petersburg Electrotechnical University (LETI), Russia
nlschegoleva@etu.ru

Abstract: This article presents a method of improving the quality of a reconstructed 3D surface of a face or its parts using the inverse distance method (IDM). The proposed method eliminates undesired holes in the range image, which appear due to significant omissions of measurements from the face scanner. Application of the method allows to build a 3D surface with the desired resolution, without solving the problem of triangulation therefore reducing the computational cost. The results of the experiments show the efficiency and the quality of the reconstructions obtained using the proposed algorithm.

Keywords: Range image, Method of inverse distance, 3D surface reconstruction, Hole, Face recognition

1. Introduction

The use of 3D face models in biometrics is becoming not only a necessity, but a reality. The former is caused by the requirement to increase the efficiency of Face Recognition System (FaReS) in real conditions of initial data reception. The latter is supported by a huge leap in the development of computer technology – increase of computer memory and speed, a more extensive use of new processing methods oriented on digital images as two-dimensional data, mathematical modeling and based on all of this – creating a wide range of methods and techniques of scanning 3D objects and receiving their digital models in 3D format.

However, as has been noted in [1, 7, 9], 3D face recognition is not completely perfect. For instance, an inaccurate illumination of a scanned face might negatively affect on the result of its 3D reconstruction. Thus, excessively light-struck illuminated facial areas, or facial regions of very poor illumination (and/or low contrast) might negatively affect on face forms reconstruction. As a result, a smooth surface might contain artifacts such as "spikes" (i.e. thorns and outgrowths), and gaps in 3D surface called “black holes” might appear in sharp forms and abrupt junctions (e.g. the tip of a nose, superciliary arches, the wings of a nose and nasolabial folds) [9, 10].

In connection with the aforementioned information this paper contains a method of qualitatively improving 3D face reconstruction for the case of “black holes” on the resultant surface.

2. The receiving initial 3D face forms process

We won’t stop on various scanning techniques. Let just mention that it might be laser scanners with assessment of distance from the scanner to the elements of object’s surface,
special scanners with structured illumination of the object’s surface and mathematical
treatment of bends of lines scanning these surfaces, or it might be scanners processing syn-
chronous stereomates of a face image by a photogrammetric method.

Step 1 – scanning. An example of facial region scanning moment by a scanner-camera
“Minolta Vivid” type is shown on the Fig. 1 under number 1 [9]. The position of the person
relative to the camera fixed for during several seconds, and his face is illuminated (using
structured illumination) and is close enough to the camera.

One should note that modern scanning systems and cameras can work without special il-
 lumination from a distance of around 3-4 meters. And use of several cameras and video
streams makes it possible to scan faces in motion.

The result of facial region scanning is a point cloud \( p_l \), where \( l = 1, 2, ..., L \), as shown in
Fig. 1, Image 2. The parameter \( L \) can vary from several hundreds to several tens of thou-
sands. Every point \( p \) is represented in 3D space by its coordinates \( \{x_l, y_l, z_l\} \), where \( x \) is a
coordinate that is perpendicular to the face symmetry axis, \( y \) is a coordinate that is parallel
to the face symmetry axis and \( z \) is a coordinate that represents the height of the point relative
to the XY plane. At the same time, physical accuracy of height (or depth) measurement
should be accurate to 1mm. As a result of the scanning process, the human face is presented
as a cloud of points, which is described by three vectors of size \( L \times 1 \) each.

Step 2 – connection points to the triangular grid. The variables \( \{x_l, y_l\} \) are unrelated co-
ordinates on the XY plane, placed on an irregular grid. In order to present the cloud as a
digital image, defining a scanned 3D form, one needs to transfer the coordinates \( \{x_l, y_l\} \) onto
a regular grid and thus recalculate the \( z_l \) coordinates. This is done on the basis of triangula-
tion procedures with straight-line interpolation. In this case every three \( z_l \) coordinates are
connected to form a triangle, the vertices of which are connected to their nearest neighbours
to form new triangles. Note that only a single plane can be drawn through any three points,
and a triangle is the simplest polygon, the vertices of which also define the sides of the tri-
gle. At the same time, no other point should lie within this triangle. The \( z_l \) coordinates are
defining the vertices of the triangles, create supporting points of the grid, the nodes of which
define the heights of the facial region relief.

Step 3 – coordinate translation. Using these triangles one can translate the coordinates
\( \{x_l, y_l\} \) to a regular grid and recount the \( z_l \) coordinates. The received values are projected
onto the regular grid orthographically. Together with the calculated new vertices \( z_l \) they
form a range image, an example of which is shown in Fig. 1, Image 4. Some transformation
methods and algorithms, as well as examples of them are presented in [3].

Step 4 – range image creation: The range image is then smoothed in all directions (using
rotation, shift, deletion and depth approximation techniques) and is represented in greyscale,
for instance. This result is shown in Fig. 1, Image 5.
3. 3D surface reconstruction via the inverse distance method

It should be noted that the triangulation and calculation of the connected $z_i$ coordinates produce very good representations of facial surfaces in 3D space and resulting range images. However, this is achieved by use of a very large number of triangles in the original grid, which inevitably leads to high computational costs. One can reduce the number of triangles (and hence reduce the computational cost) by using “adaptive triangulation”, which involves the placement of more triangles where sharp and deep transitions exist on the facial surface (i.e. at the eyebrows, eyes, nose and the area below it, mouth) and less triangles in the places that correspond to gradual changes (such as the forehead and cheekbones). Such a version is shown in Fig. 1, Image 3. In some cases it may be enough to create a triangular grid based only on those nodes (vertices) that are connected with anthropometric points. There cannot, however, be more than 80-100 points of this kind.

If the initial cloud \( \{x_i, y_i, z_i\} \) lacks a considerable amount of depth measurements from the scanner to the facial regions, the resulting triangular grid will have a large amount of polygon gaps. This leads to the occurrence of holes on the range image surface. These “holes” correspond to the information gaps which in turn prevent us from receiving exact forms of 2D facial surface. An example of these errors is shown in Fig. 2, where Images 2 and 3 have been taken from [1]. Fig. 2, Image 1 was received from range image values \( \{z_i\} \) for coordinates \( x_i \) and \( y_i \), using by a random-number generator. Thus a sort of “reverse transfer” was made from the range image to the initial data cloud.

The amount of points is 2000. Points of two different shades belong to the left and right sides of the range image. In a cloud, constructed in such a way, one can distinctly see “ragged edges” and considerable gaps (marked by ellipses), that create the “holes” on the range image.

![Figure 2. Range image reconstruction problems with incomplete baseline data](image)

The lack of data about the depth in the range image images leads to a “thorns” appearance on the reverse side of the 3D facial forms. One usually eliminates “black holes” of “range image” images with the help of a spline interpolation method in the interactive mode. Thus, the struggle with these information gaps (and the necessity of a “black holes” interactive elimination in the “range image” images) is one of the peculiarities of the measurement information triangulation method. It will be shown that use of the surface reconstruction inverse distance method (IDM) can successfully deal with the elimination of black holes from range images. If we are not interested in the face model, but in the problem of the face identification in 3D form, then we have a lot of variants of construction of all the
facial surface or only its necessary parts with their further use in a process of identification. We can also manage without the initial unification of points \( p_i \) into triangles, omitting the examination and solving the triangulation problem. Instead of that, we can base on the received measurements \( \{x_l, y_l, z_l\} \), that belong to an irregular grid of measurements, and thus construct a 3D surface with a required resolution (concerning range image – with a required size \( M \times N \) of the image). The simplest way to do that is realized in a of Kriging method based on the inverse distance method [2]

4. Description of the transformation algorithm based on IDM

We solve the problem of data transposition from an irregular grid to a regular one in 2 steps [6].

1. The initial information for the first step includes a set of points \( \{x_l, y_l, z_l\} \), represented by the corresponding vectors \( X, Y, Z \), each of \( L \times 1 \)-size.

The aim of this step is the construction of a model \( z_l = f(x_l, y_l) \), that represents the dependence of \( z_l \) on \( \{x_l, y_l\} \). To reach the aim we make the following steps.

1.1. Let us represent all coordinates \( \{x_l, y_l\} \) of the initial data by a vector \( K \) in complex form in the following way:

\[
K = X + jY,
\]

\( j = \sqrt{-1} \) \hspace{1cm} (1)

and it is assumed that the vector \( K \) do not contain data that \( k_l \neq k_d \) for \( l \neq d \). This condition of “data non-multiplicity” is the only one in solving of the given task.

1.2. Now create a matrix \( K \), repeating \( L \) times the resulting vector \( K \):

\[
K = [K K \ldots K],
\]

\( \text{where the matrix } K \text{ is of size } L \times L \)

1.3. Compute the distance between all the nodes of the irregular grid:

\[
D = \text{abs}(K - K^T),
\]

\( \text{where the matrix } D \text{ is of size } L \times L \)

Note that only under the constraints of the data non-multiplicity condition is the rank of \( D \) equal to its order. Only in this case, a transition to the next step of the algorithm.

4. Compute a vector of regression parameters \( B \) such that:

\[
B = D^{-1}Z,
\]

\( \text{where the vector } B \text{ has a size } L \times 1 \)

2. The initial information for the second step is:

- a set of points \( \{x_l, y_l, z_l\} \), recorded in corresponding vectors \( X, Y, Z \);
- a vector \( B \) containing the regression parameters;
- values of \( L, M \) and \( N \).

The aim of the second step is initial data (cloud) transfer onto a regular grid of $M \times N$ size, and as a result we will receive a range image. The calculation of each value $I(m,n)$ of the range image is calculated in the following way:

$$I(m,n) = \sum_{i=1}^{L} b_i \cdot |k(m,n) - k_i|, \quad m=1, 2, \ldots, M \quad \text{and} \quad n=1, 2, \ldots, N,$$

(5)

where: $k(m,n) = x_n + jy_m$, $\forall$ $n=1, 2, \ldots, N$ and $m=1, 2, \ldots, M$;

$|k(m,n) - k_i|$ – modulus of the difference between the new and old coordinates;

$x_n = \min(x) + n(\max(x) - \min(x))/(N-1)$;

$y_m = \min(y) + m(\max(y) - \min(y))/(M-1)$; $k_i = x_i + jy_i$.

Note that the solution (5) has also been achieved in the condition of “data non-multiplicity”, which can be written as follows: $|k(m,n) - k_i| \neq 0$, $\forall l \neq n$ and $l \neq m$.

The implementation algorithm (5) is given below.

2.1. For set $\{x_l, y_l\}$, recorded in the corresponding vectors $X$ and $Y$, calculate the coordinates of the boundary so that:

$$\begin{align*}
\max_Y &= \max(Y); \min_Y = \min(Y); \\
\max_X &= \max(X); \min_X = \min(X).
\end{align*}$$

(6)

2.2. Then calculate the sampling interval of a regular grid with considering the size $M$ and $N$ of the range image:

$$\begin{align*}
\delta Y &= (\max_Y - \min_Y)/(M-1); \quad \delta X = (\max_X - \min_X)/(N-1).
\end{align*}$$

(7)

2.3. Now define all the coordinates of a regular grid for a given size of range image and write them in the corresponding vectors:

$$\begin{align*}
X_r &= (\min_X : \delta X : \max_X); \\
Y_r &= (\min_Y : \delta Y : \max_Y).
\end{align*}$$

(8)

2.4. Create a “zero” work array of size $M \times N$ for range image:

$$I = [0]_{M \times N}.$$ 

(9)

2.5. For all the current coordinates of $m=1, 2, \ldots, M$ and $n=1, 2, \ldots, N$ form the vector $X_r, Y_r$, consisting of $L$ rows of coordinates $[X_r(n)+jY_r(m)]$, so that:

$$X_r = \begin{bmatrix}
X_r(n) + jY_r(m) \\
X_r(n) + jY_r(m) \\
\vdots \\
X_r(n) + jY_r(m)
\end{bmatrix}.$$ 

(10)

2.6. Calculate the distance between the new and original coordinates:

$$D_{new} = |X_r - K|.$$ 

(11)

2.7. Calculate the values of “the range image” for the pixel $(m,n)$:

$$I(m,n) = D_{new}^T B$$

and then select a new current value of $m$ and $n$, and repeat steps (10) – (12).
5. Experiments

5.1. The base of the face images, used in the experiment

In the experiments on research of the method of “data carries by IDM” we used images from the “Texas 3-D Face Recognition Database” [4], and images from the cited articles [9, 1, 7] as well. All these images are shown in Fig.3.

At the same time the images [4] were used to check the developed algorithm of “data carries by IDM” and to assess the accuracy of the reconstruction of derivable 3D forms of range image.

![Sample of images [9, 1, 7, 4]](image)

Figure 3. Sample of images [9, 1, 7, 4]

The images [9, 1, 7] were used for a comparative analysis of the results obtained the method of “data carries by IDM”, presented in this section, and the results, showed in the cited articles. It concerned especially the problems of initial data incompleteness and the problem of “black holes”, as in the listed articles these very problems were discussed and similar examples were given.

5.2. Reconstruction of the full Range Images image

Unfortunately, we don’t have the experimental \{x_l, y_l, z_l\} coordinates that are necessary to create an initial data cloud which would allow for the validation of the proposed method and algorithm presented above.

For all the experiments discussed below the data was created from pre-existing available originals – range images.

The procedure of the data receiving is as follows.

We obtained the values \{z_l\} of the initial data cloud from the range image for the coordinates \(x_l\) and \(y_l\), using a random uniformly distributed number generator. The coordinates \(x_l\) and \(y_l\) in this case are not whole numbers, and thus take on values on a continuous scale \(1, N\) for \(x_l\) and \(1, M\) for \(y_l\). This way of generating the initial data gives \(K >> NM\) of different pairs of coordinates \{x_l, y_l\}, corresponding to an irregular grid in that lies within the specified bounds. The \{z_l\} take whole values that exactly match the pixels of the range image with the coordinates \(x\) and \(y\), approximated to the nearest whole number. To make the values of the
properties $z_i$ of a "true measurement", a normal noise for all values $1 < z_i < \max(Z) - 1$ and a steady noise on the range limits are imposed upon a vector of the $z$ values. Noise characteristics in both cases are the parameters $(0, 1)$.

As we have the original – the initial range image, it allows us to compare the results of its reconstruction on generated data with this original.

Fig. 4, Image 1 shows the original image of size 270 by 210 pixels (range image [4]). Fig. 4, Image 2 shows the same range image represented in 3D-space.

![Figure 4](image)

Figure 4. The two forms of the original range image representation

It is interesting to note drawing lines of the nose, the eyes and the mouth in the left image (range image) and a "rough graininess" of the 3D surface in the right image. However, it is clear that the values $\{z_i\}$ of the range image quite accurately represent the height of all the points shown in the right image – i.e. on the entire 3D surface. We should also note that the range image does not contain "black holes" and, therefore, there are no thorns or undefined values in 3D form in the right image.

Now, using the generator of a uniformly distributed random numbers, form the $K \ll NM$ coordinate pairs $\{x_i, y_i\}$ and carried them to the border $1, N$ and $1, M$. The location of these coordinates on the range image for $K = 550$ is shown in Fig. 5.

![Figure 5](image)

Figure 5. Selected coordinates and results of reconstruction

Then, using the above-mentioned method, we obtained from the same image the 550 values of $\{z_i\}$ for all 550 of the generated coordinates $\{x_i, y_i\}$. To simulate more realistic values of $z_i$, we imposed noise upon a vector of the values $\{z_i\}$, using the previously discussed method. Thus, as a result of all the done actions we received the initial data cloud $\{x_i, y_i, z_i\}$, where the coordinates $\{x_i, y_i\}$ are placed on an irregular grid. Finally, we perform
a back transfer of coordinates \( \{x_l, y_l, z_l\} \) onto a regular grid by the method described in the Section 3. The results of this are shown on the right hand side of Fig. 6. We should note that we used only 550 triples of the coordinates \( \{x_l, y_l, z_l\} \) for the reconstruction of the whole image. The result rather accurately reflects the basic 3D forms of the range image. The mean-root square error was \( \approx 8 \), and the vector of the regression equation for this case includes only 550 coefficients. As applied to the reduction of the dimension of features, a decline of is more than 100 times (since \( 270 \times 210/550 \approx 103 \)).

Another way of comparing the results of the range image reconstruction is shown in Fig. 6.

![Figure 6. Results of reconstruction](image)

The top line shows both the initial and reconstructed images, and the bottom line shows the profile of these representations, constructed along the facial line of symmetry. Here, the X axis represents the line numbers of the range image and the Y axis the depth values along the line of symmetry of the range image. The thin line is the facial profile of the original image and the thick line is the facial profile of the reconstructed image.

The profiles perfectly match in the areas of the forehead, nose, mouth and chin. And we should remember that the new – reconstructed – image is based only on 550 coordinates, received randomly.

The similarity of the profile lines around the forehead, nose and chin can be used to compare two versions of the range image presented in [7, 8, 5]. In these papers propose to eliminate from consideration those parts of the facial 3D surface that may change with facial expression. They also emphasize the fact that part of the face that remains rigid provides sufficient information for its recognition.

Finally, the results of the reconstruction using the proposed method can be judged on a qualitative level. Thus, in Fig. 7 we can see: Image 1 - intensity image, Image 2 - range image corresponding, Image 3 - the result of the range image reconstruction. As one can see, the quality of the reconstruction is quite high, especially if we take into consideration that this reconstruction was performed using only on 600 points (!).
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5.3. Range image reconstruction when there are “black holes”

Now let us answer the question: is it possible to improve the quality of range image reconstruction by increasing the initial number of points $K$ in the measurement cloud?

The answer is: it is theoretically possible but, in practice, difficult to implement.

Indeed, the greater the number of the points in the measurement cloud is, the better you can reconstruct the facial 3D surface. But in practice we will face the following features:

– when calculating the regression parameters using equation (4), one should invert the distance matrix $D$, the degree of which is directly related to the number of points $K$ of the measurement cloud;

– if $K<1000$, that usually happens in the reconstruction task simulation, there are no problems with the inversion of a distance matrix $D$. But the generation of $K$ triples of coordinates $\{x_l, y_l, z_l\}$ for $K \approx 1000$ and providing data non-multiplicity within the limited size $M$ and $N$ can be challenging. When a face is scanned (on a physical layer), usually $K>>1000$, and then the discussed class of problems cannot be solved by the proposed IDM. In this case, using reconstruction based on triangulation.

But we also know that the triangulation method cannot be used effectively in the cases when the physical scanning of separate facial areas is accompanied by the omission of measurement data for these areas. In this case “black holes” appear on the range image surface.

“Patching up those holes” is based, as is known, on the method of spline interpolation, which is beyond the scope of the triangulation method. Here again we use the method of reconstruction, based on IDM. Let us analyze this in details.

So, our goal is to reconstruct the facial 3D surface using an initial data cloud. First of all, we will assume that the facial 3D surface has no vertical (i.e., along the axis $Z$) elevation changes and, therefore, we cannot find in the data cloud any data with coordinate values $\{x_l, y_l, 0\}$ for any $l \in L$.

Based on this assumption, any coordinates of values $z_l \approx 0$ are excluded from further consideration. Then we exclude from consideration the coordinates that do not comply with the requirement of data non-multiplicity. So, we will make the range image reconstruction using IDM on the remaining data.

Let us analyze the final and transitional results of some tests of the solution.
An example of the original image (range image) with black holes is shown in the Fig. 8, Image 1 [1].

Holes on the range image occur as isolated local black areas. There are only six big holes in total (i.e. two in the centre of the nose, two around the eyes and two in the area above the eyebrows) and 6 small holes. On the original range image a vertical line has been drawn along the axis of symmetry of the face, along which the profile is constructed. In Fig. 8, Image 2 we can see the location of the coordinates \( \{ x_l, y_l \} \) on the XY plane, generated using a random number generator. Note that some of these points lie within the black holes. Fig. 8, Image 3 shows the result of range image reconstruction based on the \( \{ x_l, y_l, z_l \} \) coordinates, where \( z_l \neq 0 \) and which satisfy the requirement of data non-multiplicity. In this experiment the number of these coordinates is not more than 300. The left hand side of the bottom line of the Fig. 8 shows a 3D representation of the original image, with the edges of the visible black holes outlined in black.

![Figure 8. The original image (range image) and the result of its reconstruction](image)

The right-hand side shows the result of the range image 3D reconstruction, where we can clearly see a human profile (the nose and upper lip area, eye holes, forehead) that was not visible in the left image.

We presented a qualitative aspect of the reconstruction above, and now let us present a quantitative aspect. The left-hand side of Fig. 9 shows the profile along the line of symmetry of the original range image.

![Figure 9. Profiles on the symmetry line for range image and the reconstruction result](image)

Here the X axis contains the line numbers of the range image and the Y axis the depth value at the respective points along the line of symmetry of the range image. The sudden profile drop in the left image (the area from the line 115 to the line 135) is due to the largest
“black hole“, which is located right in the centre of the tip of the nose in the original range image.

On the right-hand side of Fig. 9 we can see the profile obtained along the line of symmetry of the range image reconstruction (thick red line), imposed onto the original profile (thin line). We see that the profiles along the line of symmetry coincide almost everywhere, except for the one differential region (“black hole“ area). In this very place the missing range image surface was reconstructed, and the thick line was outlines the reconstructed (imaginary) tip of the nose. The mean-root square error in the profiles in this case did not exceed 35.

In evaluating the results, following should be noted:
First, it is evident that the result of the range image reconstruction does not contain that amount and that the quality of the “black holes” that were on the original image.
Second, on the reconstructed 3D form we can clearly see a human profile (the nose, the eye pits, the forehead), that was not visible in the original range image.
Finally it is clear that the facial form by a symmetry line is fully recovered as a result of the using of the reconstruction method based on IDM. This is an advantage of range image reconstruction using IDM and demonstrates the possibility of its use for practical purposes.
As we managed to achieve this kind of the range image reconstruction quality with $K = 300$, $M = 215$ and $N = 95$, we can assume that the proposed method has very good characteristics.

Recall that the entire reconstruction process involves the following steps:

− the generation of the coordinates $\{x_l, y_l\}$;
− the determination of $\{z_l\}$ for the coordinates $\{x_l, y_l\}$ on the original range image;
− removal of coordinates $\{x_l, y_l, z_l\}$, where $z_l \approx 0$;
− verification of the conditions of non-duplicate data and the removal of any duplicate data;
− the parameters of the regression equation calculation;
− the construction of a new surface range image.

The time spent on both range image reconstruction together and the display of transitional graphic results (see Fig. 8 and Fig. 9), was no more than 8 seconds using a Pentium Dual-Core CPU T4200 computer with a clock speed of 2 GHz and 3GB of RAM.

To conclude the section, Fig. 10 presents in graphical form the comparative results of range image reconstruction for two variants of a measurement cloud, with $K = 299$ and $K = 248$.

Values of a parameter $K$ are not round numbers (not 300 and not 250) due to the elimination of duplicate data. In the first variant the number of multiple pairs of coordinates $\{x_l, y_l, z_l\}$ is 1, in the second variant we have 3 pairs of coordinates of that kind.

Above the graphs in Fig. 10 there is also an assessment of the root-mean-square difference profiles along the symmetry lines for the original and the new range image.

![Figure 10. Range image reconstruction results for K = 299 and K = 248](image-url)
We can see that the errors are not connected with the number of points but with a form of the 3D reconstructed image. But we should remember that for every variant we used a separate generator of coordinates \( \{x_i, y_i\} \), that gave some difference in the results.

5.4. Reconstruction of parts of the range image

The most informative part of a human face is the central part, which covers the eyes, the eyebrows and the nose. This part of the face is the least susceptible to changes during a conversation, or due to either hair-style changes or a false beard or moustache. This feature of the selected part is widely used in solving problems of people face recognition and in building 3D face models. An analysis of such face recognition problem can be found, for instance, in [1]. In this connection we will show how well this part of the face can be reconstructed in the form of range image using on IDM.

Thus, Fig. 11 shows: Image 1, 2 – the range image in «gray» and «hsv» formats; Image 3 – a profile along the symmetry line of the selected part of the face. In the image, which presented in a «hsv» format, clearly drawn forms are the same and different levels of height.

![Figure 12. Range image original image and profile along the line of symmetry](image)

As you can see, the reconstructed range image accurately represents the original. At the same time the same height levels are seen both in the obtained and original images. A profile of the reconstructed image also accurately covered the original profile. The mean-root square error was 8 (using 256 levels of brightness).

Fig. 12 shows the results of range image reconstruction using 599 points. Here Image 1 and Image 2 shows the range image in «gray» and «hsv» formats; Image 3 shows the profile along the symmetry line of the selected part of the face, superimposed onto the profile of the original range image.

![Figure 12. Range image Reconstruction and profile on the symmetry line](image)

The variation of the mean-root square error in relation to the number of points in the initial data cloud is shown in table below. Here the first row shows the number of points, and the second the mean-root square error. We can see that if \( K \geq 300 \) the error level varies between 6 and 8.
As we know from [7, 8, 5], the nose is the least susceptible part of the face to changes due to different facial expressions (such as anger, joy, etc.). At the same time the nose as a 3D form is one of the most characteristic elements of a person. Therefore, identification of people by their noses has become a popular subject matter in practical biometrics. Using the discussed method we can also reconstruct a nose in 3D form (or as a range image) easily. Let us show this through a simple example.

![Figure 13. Input data and results of reconstruction range image](image1)

**Fig. 13** shows: in Image 1 – the original image (i.e. the range image), in Image 2 – an artificially made hole on the tip of the nose, in Image 3 – the \(\{x_l, y_l\}\) coordinates that determine the value \(z_l\), and in Image 4 - the result of nose reconstruction using 500 points.

The Fig. 14 shows a nose 2 (on the left) and a nose 4 (on the right) from Fig. 13, both presented as 3D form. In the left image there is a marked spot which corresponds to a black hole in the original nose image, while in the right image the hole is missing. It should be noted, that this is a result of a procedure of the range image reconstruction using IDM.

![Figure 14. The original and reconstructed nose in 3D form](image2)

**Figure 14.** The original and reconstructed nose in 3D form

### 6. Conclusions

This paper presented a method of improving the quality of a reconstructed 3D surface of a face or its parts using the inverse distance method (IDM). The experiments demonstrated that the result of range image reconstruction does not contain the amount or quality of black holes that were present in the original image. On the reconstructed 3D representation, we can clearly see a human profile that was not visible in the original range image. The dis-
cussed method has a lower computational complexity. This advantage of range image reconstruction using IDM demonstrates the possibility of its use for practical purposes.

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